NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

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| **QUALIFICATION:** Bachelor of Science honours in Applied Statistics | |
| **QUALIFICATION CODE:** 08BSSH | **LEVEL:** 8 |
| **COURSE CODE:** STP801S | **COURSE NAME:** STOCHASTIC PROCESSES |
| **SESSION:** JUNE 2022 | **PAPER:** THEORY |
| **DURATION:** 3 HOURS | **MARKS:** 100 |

| FIRST OPPORTUNITY EXAMINATION QUESTION PAPER | |
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| **EXAMINER** | Prof. RAKESH KUMAR |
| **MODERATOR:** | Prof. PETER NJUHO |

## INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

**THIS QUESTION PAPER CONSISTS OF 3 PAGES** (Including this front page)

**Question 1. (Total marks: 10)**

**(a)** What is a stochastic process? (2 marks)
**(b)** Classify the stochastic processes according to parameter space and state-space using suitable examples. (8 marks)

**Question 2. (Total marks: 10)**

**(a)** Define martingale. (2 marks)
(b) Differentiate between super- and sub-martingales. (3 marks)
(c) What is gambler's ruin problem? (5 marks)

**Question 3. (Total marks: 20)**

**(a)** Show that the transition probability matrix along with the initial distribution completely specifies the probability distribution of a discrete-time Markov chain. (10 marks)
**(b)** Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that probability of a rainy day following a dry day is 1/2. Develop a two-state transition probability matrix of the Markov chain. Given that May 1, 2022 is a dry day, find the probability that May 3, 2022 is a dry day. (10 marks)

**Question 4. (Total marks: 10)**

**(a)** Differentiate between persistent and transient states. (3 marks)
**(b)** Classify the states of the Markov chain whose transition probability matrix is given below: (7 marks)

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

**Question 5. (Total marks: 10)**

**(a)** Find the steady-state probabilities of the Markov chain whose one-step transition probability matrix is given below: (8 marks)

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

**(b)** What is stationary distribution of a Markov chain? (2 marks)

**Question 6. (Total marks:20)**

**(a)** What is a Poisson process? (5 marks)
**(b)** Suppose that the customers arrive at a service facility in accordance with a Poisson process with mean rate of 3 per minute. Then find the probability that during an interval of 2 minutes:
(i) exactly 4 customers arrive (ii) greater than 4 customers arrive
(iii) less than 4 customers arrive
( $e^{-6}$ =0.00248) (10 marks)
**(c)** Prove that if the arrivals occur in accordance with a Poisson process then the interarrival-times are exponentially distributed. (5 marks)

**Question 7. (Total marks: 20)**

**(a)** Derive the Chapman-Kolmogorov equations for continuous-time Markov chain. (10 marks)
**(b)** Derive Kolmogorov forward differential equation. (10 marks)

---------------------------------------END OF QUESTION PAPER.........................................................